Find the area of the shaded region.
1)

2)

3)

4)


Sketch the region enclosed by the given curves. Decide whether to integrate with respect to $x$ or $y$. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.
5) $y=x+1, \quad y=9-x^{2}, \quad x=-1, \quad x=2$

6) $y=x, \quad y=x^{2}$

7) $y=x^{2}, \quad y=x^{4}$

8) $y=\frac{1}{x}, \quad y=\frac{1}{x^{2}}, \quad x=2$

9) $y=x^{2}, \quad y^{2}=x$

10) $y=x, \quad y=\sqrt[3]{x}$

11) $y=x^{3}-x, \quad y=3 x$

12) $y=x^{2}, \quad y=\frac{2}{x^{2}+1}$

13) $y=\sqrt{x}, \quad y=\frac{1}{2} x, \quad x=9$

14) $y=8-x^{2}, \quad y=x^{2}, \quad x=-3, \quad x=3$

15) $y=\cos x, \quad y=\sin 2 x, \quad x=0, \quad x=\frac{\pi}{2}$

16) $x=2 y^{2}, \quad x+y=1$

17) $x=1-y^{2}, \quad x=y^{2}-1$

18) Use calculus to find the area of the triangle with the given vertices: $(0,0),(2,1),(-1,6)$

19) Use the Midpoint Rule with $n=4$ to approximate the area of the region bounded by the given curves:

$$
y=\sqrt[3]{16-x^{3}}, \quad y=x, \quad x=0
$$



Use a graphing calculator to find approximate $x$-coordinates of the points of intersection of the given curves. Then find (approximately) the area of the region bounded by the curves.
20) $y=x^{2}, \quad y=2 \cos x$
21) $y=x \cos \left(x^{2}\right), \quad y=x^{3}$
22) The curve with equation $y^{2}=x^{2}(x+3)$ is called Tschirnhausen's cubic. If you graph this curve you will see that part of the curve forms a loop. Find the area enclosed by the loop.
23) Find the area of the region bounded by the parabola $y=x^{2}$, the tangent line to this parabola at $(1,1)$, and the $x$-axis.

24) Find the number $b$ such that the line $y=b$ divides the region bounded by the curves $y=x^{2}$ and $y=4$ into two regions with equal area.
25) Find the values of $c$ such that the area of the region enclosed by the parabolas $y=x^{2}-c^{2}$ and $y=c^{2}-x^{2}$ is 576.

